

Meta-Learning for Time Series Forecasting Ensemble *

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Abstract

Amounts of historical data collected increase together with business intelligence applicability and demands for automatic forecasting of time series. While no single time series modeling method is universal to all types of dynamics, forecasting using ensemble of several methods is often seen as a compromise. Instead of fixing ensemble diversity and size we propose to adaptively predict these aspects using meta-learning. Meta-learning here considers two separate random forest regression models, built on 390 time series features, to rank 22 univariate forecasting methods and to recommend ensemble size. Forecasting ensemble is consequently formed from methods ranked as the best and forecasts are pooled using either simple or weighted average (with weight corresponding to reciprocal rank). Proposed approach was tested on 12561 micro-economic time series (expanded to 38633 for various forecasting horizons) of M4 competition where meta-learning outperformed Theta and Comb benchmarks by relative forecasting errors for all data types and horizons. Best overall results

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were achieved by weighted pooling with symmetric mean absolute percentage error of 9.21% versus 11.05% obtained using Theta method.

Keywords: univariate time series model; forecasting ensemble; meta-learning; random forest; M4 competition.

1 Introduction

Forecasting of key performance indicators as well as any important dynamics in any organisation should be business intelligence task of high priority, where the aim is to envisage indicator's values into the future, based on historical observations. An accurate forecast mitigates uncertainties about future outlook and can help to reduce errors in decisions and planning, which directly influences achievability of goals and contributes to risk management. Forecasting should be an integral part of the decision-making activities in management (Hyndman, 2010), since strategic success of the organisation depends upon effective relation between accuracy of forecast and flexibility of resource allocation plan (Wacker and Lummus, 2002). It is expected that increasing amounts of historical data records, which constitute useful resources for the forecasting task, will facilitate accurate forecasting as well as boost importance of these forecasts.

Almost 40 years ago worldwide Makridakis forecasting competitions have started (organized in 1982, 1993, 2000, 2018 and 2020) with a goal to benchmark progress in forecasting techniques and derive scientific insights in time series forecasting. During these events teams of participants compete in obtaining forecasts for increasing amounts of time series from diverse fields and results are summarized into recommendations on usefulness of various time series models or their ensembles. In the recent M4 competition (Makridakis et al., 2018) the leading forecasting techniques (12 from 17 most accurate ones) featured model ensembles which pool forecasts of several mainly statistical models. The best solution was submitted by Uber Technologies where hybrid technique combined statistical forecasting model with neural network architecture. The next most successful submission (Montero-Manso et al., 2020) featured ensemble of statistical models where weights were tuned and these weight recommendations were learned by machine learning model for later prediction. Insights after an older M3 competition (Makridakis and Hibon, 2000) could be summarized succinctly as follows - forecasts from univariate time series models almost always (except for annual data) are more accurate than forecasts from multivariate time series models with external variables (i.e. macroeconomic indicators) and comparison between univariate approaches revealed that more complex models don't guarantee higher accuracy.

Proceeding from results of Makridakis competitions (Makridakis et al., 2018; Makridakis and Hibon, 2000) and numerous academic researches (Clemen, 1989; Hansen and Salamon, 1990; Hendry and Clements, 2004; Timmermann, 2006; Kolassa, 2011) it can be concluded that ensemble of univariate time series models often outperform the best member of the ensemble with respect to forecasting accuracy. Success of forecasting ensemble lies in the diversity of its members (Oliveira and Torgo, 2015), which contributes to robustness against concept drift (Zang et al., 2014) and enhances algorithmic stability (Zou and Yang, 2004). Besides ensemble diversity, individual accuracy of its members is also of utmost importance (Lemke and Gabrys, 2010). Regarding strategies when combining forecasts from several models, simple arithmetic average with all members weighted equally often outperforms more complex approaches which seek to find optimal weights, for example, based on model's Akaike information criterion (Kolassa, 2011) or model's in-sample forecasting errors (Smith and Wallis, 2009) on the recent dynamics of time series in question. In practice, to avoid corrupting final forecast by a single inaccurate model, variants of robust average or simply median are recommended in forecast pooling (Hendry and Clements, 2004). Choice of weight for ensemble member often relies upon in-sample forecasting errors, although when approximate ranking of model pool is available instead of exact errors, weight could be derived from model's reciprocal rank (Aiolfi and Timmermann, 2006).

Machine learning and optimization fields have a "no free lunch" rule, which states that if a method outperforms other methods on a specific data then there exists data where this method is outperformed by others, i.e. there exists no forecasting method that performs best on all types of time series (Bauer et al., 2020). It can be deduced that the same rule should hold if instead of a single method we use a more complex technique - an ensemble of a fixed size with members weighted equally. Efforts to adjust ensemble size, choice of members or their weights could potentially overcome this rule by forming ensemble adaptively based on dynamics we try to extrapolate into the future.

Our research explores an adaptive construction of forecasting ensemble, consisting of various statistical as well as a few machine learning methods, with the help of meta-learning, which seeks to rank a pool of methods and recommend ensemble size based on characteristics of historical time series data. Recommendations of introduced forecasting assistants are based on training regression meta-models through forecasting experiments on a diverse set of real world examples - micro-economic time series from M4 competition. Experiments compare introduced forecasting ensemble based on recommendations from assistants with the best benchmark methods from M4 competition - Theta and Comb, which were outperformed only by 17 out of 49 submissions in M4 competition (Makridakis et al., 2018; Makridakis et al., 2020).

2 Related work

The most similar research with respect to our idea of forecasting assistants, after early expert system with rules derived by human analysts in (Collopy and Armstrong, 1992), are forecasting techniques based on meta-learning (Lemke and Gabrys, 2010; Talagala et al., 2018; Bauer et al., 2020) and recommendation rules (Wang et al., 2009; Zuefle et al., 2019). In general, meta-learner after induction phase is capable to indicate which learning method is the most appropriate to a given problem (Rokach, 2006). Meta-learning concept for time series forecasting uses machine learning model (i.e. decision tree or ensemble of trees) and trains it on a set of features, – various characteristics of time series, – with a goal of recommending the most suitable univariate time series model. It was also found that meta-learning is effective even when the meta-learner is trained on time series from one domain and tested on time series from another domain (Ali et al., 2018), suggesting machine learning universal capability more widely known as transfer learning. FFORMS (Feature-based FORecast Model Selection) (Talagala et al., 2018) idea was implemented in R package *seer* besides participation in M4 competition, but due to mediocre performance it was further developed into adaptive forecasting ensemble FFORMA (Feature-based FORecast Model Averaging) (Montero-Manso et al., 2018; Montero-Manso et al., 2020), available in R package *M4metalearning*, achieving 2nd place in M4 competition. Three novel approaches for forecasting method recommendation, where meta-learning task was based on classification or regression or both, were evaluated in (Bauer et al., 2020) with recommendation considering explicitly a machine learning-based regressor method instead of a statistical one.

The main difference of FFORMA approach over FFORMS and other forecasting method recommendation systems referenced above is that combination weights for a pool of models in an ensemble is recommended instead of a single best model. Building upon success of FFORMA we propose to simplify meta-learning by decomposing it into two separate regression tasks, where A1 assistant ranks the pool of potential time series models and A2 assistant recommends ensemble size to cap the ranked list.

3 Methodology

Assistants A1 and A2 use time series features for modeling and meta-learning target attribute, which corresponds to a rank of a specific forecasting technique for A1 and a recommended ensemble size for A2. Each time series used in meta-learning is split into train/test parts and features are calculated on training part while forecasts using a pool of univariate forecasting models are obtained on testing part. Forecasting errors on testing part are estimated and forecasting models are ranked for A1 as well as all possible ensembles are

evaluated for A2.

3.1 Time series features

Table 1: Overview of time series features for meta-learning: 130 time series characteristics in total.

Feature set	Size	R package(-s) used // detailed list of feature names
catch22	22	<i>catch22</i> // DN.HistogramMode_5, DN.HistogramMode_10, CO.f1ecac, CO.FirstMin_ac, CO.HistogramAMI.even_2_5, CO.trev_1_num, MD.hrv.classic.pnn40, SB.BinaryStats.mean.longstretch1, SB.TransitionMatrix_3ac.sumdiagcov, PD.PeriodicityWang.th0_01, CO.Embed2.Dist.tau.d.expfit.meandiff, IN.AutoMutualInfoStats_40.gaussian_fmmi, FC.LocalSimple.mean1.tauresrat, DN.OutlierInclude.p_001.mdrmd, DN.OutlierInclude.n_001.mdrmd, SP.Summaries.welch.rect.area_5_1, SB.BinaryStats.diff.longstretch0, SB.MotifThree.quantile.hh, SC.FluctAnal_2.rsrangeft_50_1.logi.prop.r1, SC.FluctAnal_2.dfa_50_1_2.logi.prop.r1, SP.Summaries.welch.rect.centroid, FC.LocalSimple.mean3.stderr
tsfeats	13	<i>tsfeatures</i> // stability, lumpiness, crossing.points.fraction, flat.spots.fraction, nonlinearity, ur.kpss, ur.pp, arch.lm, ACF1, ACF10.SS, ACF.seas, PACF10.SS, PACF.seas
stlfeats	12	<i>tsfeatures</i> // nperiods, seasonal.period, trend, spike, linearity, curvature, e_acf1, e_acf10, seasonal.strength, peak, trough, lambda
hctsa	13	<i>tsfeatures</i> // embed2.incircle.1, embed2.incircle.2, ac_9, firstmin_ac, trev_num, motiftwo.entro3, walker.propcross, std1st.der, boot.stationarity.fixed, boot.stationarity_ac2, histogram.mode_10, outlierinclude.mdrmd, first.acf.zero.crossing
heterogeneity, portmanteau	6	<i>tsfeatures</i> , <i>WeightedPortTest</i> // arch.acf, garch.acf, arch.r2, garch.r2, lag1.Ljung.Box, lagF.Ljung.Box
stationarity, normality	10	<i>tseries</i> , <i>stats</i> , <i>nortest</i> // ADF, KPSS.Level, KPSS.Trend, PP, ShapiroWilk, Lilliefors, AndersonDarling, Pearson, CramerVonMises, ShapiroFrancia
kurtosis, skewness, misc	11	<i>PerformanceAnalytics</i> // kurtosis.fisher, kurtosis.sample, skewness.fisher, skewness.sample, skewness.variability, skewness.volatility, skewness.kurtosis.ratio, misc.smoothing.index, misc.Kelly.ratio, misc.drowpdown.average.depth, misc.drowpdown.average.length
Hurst	17	<i>PerformanceAnalytics</i> , <i>longmemo</i> , <i>tsfeatures</i> , <i>liftLRD</i> , <i>pracma</i> , <i>fractal</i> // PerformanceAnalytics, Whittle, HaslettRaftery, lifting, pracma.Hs, pracma.Hrs, pracma.He, pracma.Hal, pracma.Ht, fractal.spectral.lag.window, fractal.spectral.wosa, fractal.spectral.multitaper, fractal.block.aggabs, fractal.block.higuchi, fractal.ACVF.beta, fractal.ACVF.alpha, fractal.ACVF.HG
fractality	7	<i>fractaldim</i> // HallWood, DCT, wavelet, variogram, madogram, rodogram, periodogram
entropy	9	<i>TSEntropies</i> , <i>ForeCA</i> // TSE.approximate, TSE.fast.sample, TSE.fast.approx, spectral.smoothF.wosa, spectral.smoothF.direct, spectral.smoothF.multitaper, spectral.smoothT.wosa, spectral.smoothT.direct, spectral.smoothT.multitaper
anomaly	10	<i>pracma</i> , <i>anomalize</i> // fraction.TukeyMAD, twitter.iqr.fraction, twitter.iqr.infraction.pos, twitter.iqr.fraction.pos, twitter.iqr.abs.last.pos, twitter.iqr.rel.last.pos, twitter.iqr.infraction.neg, twitter.iqr.fraction.neg, twitter.iqr.abs.last.neg, twitter.iqr.rel.last.neg

Feature engineering for assistant models consisted of pooling various known time series characteristics into a collection of 130 features (see Table 1). Almost half of all features were previously introduced as state-of-the-art time series features, available in R packages *catch22* (Lubba et al., 2019), consisting of carefully

selected 22 features, and *tsfeatures* (Hyndman et al., 2019), consisting of 42 features also used in FFORMA framework (Montero-Manso et al., 2018; Montero-Manso et al., 2020).

Seeking to extend characteristics of time series we considered calculating features not only on the original data (*orig*), but also on the results of 2 transformations (*diff* and *log*):

- *diff* - first differences help to improve/achieve stationarity;
- *log* - logarithmic transform has variance stabilizing properties.

Calculating 130 features on 3 variants of time series (*orig*, *diff* and *log*) results in a final set of 390 features, which are later used for building assistant meta-learners.

3.2 Forecasting models

Representative pool of 22 univariate time series forecasting models was selected (see Table 2). Diversity of models to consider for a potential ensemble varies from simple such as seasonal naive and linear trend to complex such as BATS and Prophet models, but most of them are statistical in nature, with the exception of machine learning approaches NNAR and xgb. Model implementations from 6 R packages were used, where parameters when creating model on time series training part were chosen automatically if model implementation had that capability.

3.3 Forecasting errors

After fitting univariate time series model on training part forecasting can be performed for a required number of steps ahead, i.e. forecasting horizon. Comparison of forecasted values with ground truth allows to evaluate how accurate the forecast was and for this purpose forecasting errors are used. We estimate 3 absolute and 3 relative forecasting errors.

Absolute forecasting errors considered:

- RMSE - root mean squared error;
- MAE - mean absolute error;
- MDAE - median absolute error.

Table 2: Selected pool of 22 base models for univariate time series forecasting. Most models are statistical, except for NNAR and xgb, which are based on machine learning. The horizontal line separates a few simple models from the remaining complex ones.

Model	R package::function	Description
SNaive	forecast::snaive	Seasonal naïve method
LinTrend	forecast::tslm	Linear trend
LinTrendSeason	forecast::tslm	Linear trend with seasonal dummies
QuadTrend	forecast::tslm	Quadratic trend
QuadTrendSeason	forecast::tslm	Quadratic trend with seasonal dummies
TSB	tsintermittent::tsb	Teunter-Syntetos-Babai method (based upon Croston for intermittent demand) with optimized parameters (Kourentzes, 2014)
ARIMA	forecast::auto.arima	Autoregressive integrated moving average (Hyndman and Khandakar, 2008)
SARIMA	forecast::auto.arima	Seasonal autoregressive integrated moving average (Hyndman and Khandakar, 2008)
ETS	forecast::ets	Family of exponential smoothing state space models (Hyndman et al., 2002; Hyndman et al., 2008)
HoltWinters	stats::HoltWinters	Holt-Winters filtering with additive seasonality (Winters, 1960)
Theta	forecast::thetaf	Theta method - simple exponential smoothing with drift (Assimakopoulos and Nikolopoulos, 2000)
STL-ARIMA	forecast::stlm	ARIMA model on seasonal decomposition of time series (Cleveland et al., 1990)
STL-ETS	forecast::stlm	ETS model on seasonal decomposition of time series (Cleveland et al., 1990)
StructTS	stats::StructTS	Basic structural model - local trend with seasonality (Durbin and Koopman, 2012)
BATS	forecast::tbats	Exponential smoothing with Box-Cox transform, ARMA errors, trend and complex seasonality (Livera et al., 2011)
Prophet	prophet::prophet	Decomposable time series and generalized additive model with non-linear trends (Taylor and Letham, 2017)
NNAR	forecast::nnetar	Neural network with a hidden layer and lagged inputs (Hyndman and Athanasopoulos, 2018)
xgb-none	forecastxgb::xgbar	Extreme gradient boosting model with lagged inputs (Ellis, 2016)
xgb-decompose	forecastxgb::xgbar	Extreme gradient boosting model with lagged inputs and decomposition-based seasonal adjustment (Ellis, 2016)
thief-ARIMA	thief::thief	Temporal hierarchical approach with ARIMA at each level (Athanasopoulos et al., 2017)
thief-ETS	thief::thief	Temporal hierarchical approach with ETS at each level (Athanasopoulos et al., 2017)
thief-Theta	thief::thief	Temporal hierarchical approach with Theta at each level (Athanasopoulos et al., 2017)

Relative forecasting errors considered:

- SMAPE - symmetric mean absolute percentage error;
- MAAPE - mean arctangent absolute percentage error (Kim and Kim, 2016);
- MASE - mean absolute scaled error (Hyndman and Koehler, 2006).

Final ranking of forecasting models was constructed by averaging individual rankings, obtained for each error type separately. In case of ties after averaging out a faster model was given priority in the final ranking.

Incorporation of both absolute and relative errors into the final ranking allows to sort out models more comprehensively and without less bias towards a single type of error. Relative forecasting errors were also reported in experiments to compare introduced approach to benchmark methods (Theta and Comb).

3.4 Meta-learner model

Meta-learner for our experiments was random forest (RF) (Breiman, 2001) regression machine learning model. RF is an ensemble of many (*ntrees* in total) CART (classification and regression tree) instances, where each CART is built on an independent bootstrap sample of the original dataset while selecting from a random subset (of size *mtry*) of features at each tree node. Fast RF implementation in R package *ranger* (Wright and Ziegler, 2017) was chosen, which, conveniently for the specifics of our assistants, allows to always include some variables as candidates for binary node split besides *mtry* randomly selected ones. Time series features were left for random selection, but a few meta-information features were set to *always.split.variables*. Meta-information features were forecasting horizon length and data type (daily, weekly or monthly) and A1 assistant additionally included model name (first column in Table 2) and three dummy indicators on model capabilities such as seasonality, complexity and decomposition.

RF size *ntrees* was fixed at 256, as recommended in literature (Oshiro et al., 2012; Probst and Boulesteix, 2017). Classical RF should be composed of unpruned CART, which allows growing trees to maximal possible depth and would correspond to *min.node.size*=1 setting, but in our case *min.node.size* parameter was tuned together with *mtry* using Bayesian optimization in R package *tuneRanger* with 21 warm-up and 9 tuning iterations. Minimization objective for A1 assistant was out-of-bag root mean squared logarithmic error - a variant of RMSE penalizing errors at lower values and achieving that prediction of best ranked cases is more precise than worse ranked candidates. Minimization objective for A2 assistant was a simple out-of-bag RMSE metric. Both being RF regression models, A1 could be nick-named as "ranker" whereas A2 as "capper" due to different tasks they are dedicated to: A1 ranks 22 forecasting models to find best candidates for time series at hand while A2 tries to propose an optimal size of forecasting ensemble, i.e. number of best ranked models to choose for forecast pooling.

Two variants of forecast pooling, namely, Simple (arithmetic average) and Weighted (weighted average) were evaluated for meta-learning. A1 assistant was identical in both variants, but A2 was constructed separately after evaluation of cumulative pooling of forecasts from the best A1-ranked univariate time series models, where pooling was done either with equal weights or weights derived from reciprocal rank (Aiolfi and Timmermann, 2006).

4 M4-micro dataset

From an original M4 subset of 12563 micro-economic time series we excluded 2 monthly cases (ID=19700 and ID=19505) due to the lack of dynamics. Among 12561 selected cases level of aggregation was as follows: 1476 daily, 112 weekly and 10973 monthly. All selected cases were pre-processed by segmenting them to have a proper train/test splits at several forecasting horizons. Forecasting horizons with varying number of steps ahead were considered: 15, 30, 90, 180, 365 and 730 days for daily data; 4, 13, 26, 52, and 104 weeks for weekly data; 6, 12, 24, 60 and 120 months for monthly data.

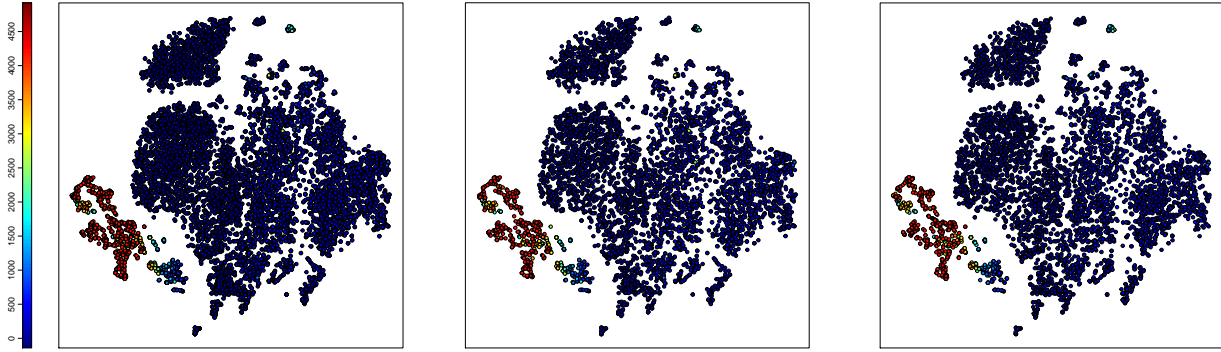


Figure 1: Visualization of M4-micro dataset by 2D t -SNE projection of time series features: full sample of 12561 time series (**left**) and result after balanced sampling into 2 cross-validation folds, containing 6281 (**center**) and 6280 (**right**) time series. Color of the point denotes the length of time series.

With respect to specifics of time series representation space (Spiliotis et al., 2020) and to avoid potential concept drift situation if meta-learners are built using random sub-spaces we split dataset by performing a careful stratified 2-fold cross-validation (2-fold CV). Stratification is done here by efficient balanced sampling (Deville and Tillé, 2004) on 3D t -SNE (van der Maaten, 2014) projection of time series features, which allows to split time series dataset into two equally-sized parts where in each part the overall representation space of initial dataset is covered thoroughly. Result of such stratification is visualized in Fig. 1, where resulting CV folds are depicted after 2D t -SNE projection.

Time series expansion by segmenting data into several train/test hold-out splits was done as follows: initially we expand a set of time series from 12561 to 38633 (see initial expansion in Table 3) to be able to test various forecasting horizons, then we increase amount of time series from 38633 to 92846 (see final expansion in Table 3) by considering additional splitting time series in half to be able to train meta-learners on more data. Initial expansion was carried out for the purpose of benchmarking various forecasting horizons and only recent data was split-off for testing, whereas final expansion was considered as a way of data augmentation to harvest more training data for meta-learners where besides initial expansion extra two splits were done where

Table 3: Expanding M4 micro dataset for benchmarking and augmentation. Expansion was performed by segmenting each time series into various train/test splits fulfilling 80/20 heuristic, which ensures that amount of data available for time series model training is at least 4 times larger than for testing, where the amount of data for testing is defined by forecasting horizon.

Horizon	Initial expansion for benchmarking			Final expansion for augmentation		
	Full sample	CV fold 1	CV fold 2	Full sample	CV fold 1	CV fold 2
15 days	1476	736	740	4374	2178	2196
30 days	1443	720	723	4201	2100	2101
90 days	1335	667	668	3699	1853	1846
180 days	1181	593	588	3313	1661	1652
365 days	1047	524	523	2693	1346	1347
730 days	780	389	391	780	389	391
\sum	7262	3629	3633	19060	9527	9533
4 weeks	112	58	54	206	106	100
13 weeks	112	58	54	206	106	100
26 weeks	47	24	23	141	72	69
52 weeks	47	24	23	119	60	59
104 weeks	36	18	18	76	40	36
\sum	354	182	172	748	384	364
6 months	10973	5486	5487	32904	16452	16452
12 months	10973	5486	5487	22317	11166	11151
24 months	5574	2792	2782	14110	7055	7055
60 months	3432	1713	1719	3630	1813	1817
120 months	65	31	34	77	33	44
\sum	31017	15508	15509	73038	36519	36519
\sum	38633	19319	19314	92846	46430	46416

possible - hold-out on older and newer halves of time series. Following the recommendation of (Cerqueira et al., 2020) for using out-of-sample holdout split in multiple testing periods, we consider 80/20 as a sufficient train/test ratio and refuse to segment time series if the amount of training data, after leaving out last few observations for testing (based on the size of forecasting horizon), drops down below 80%.

5 Experimental results

Forecasting experiment was performed using 2-fold CV by creating assistants on CV fold 1 of final expansion and testing success of assistant-recommended forecasting ensemble on CV fold 2 of initial expansion and vice versa. Besides forecasting using proposed approach benchmark methods Theta and Comb were used on initial expansion and relative forecasting errors were calculated for comparison.

Results by SMAPE (see Table 4) demonstrate that both Simple and Weighted variants of meta-learning outperform Theta and Comb techniques. Weighted slightly outperformed Simple variant for more than half (10 out of 16) horizons and also overall (see last row in Table 4) .

Table 4: Forecasting results according to SMAPE forecasting error. \sum denotes results over all horizons, the best result for each row is formatted in italic-bold.

Horizon	Theta	Comb	Simple	Weighted
15 days	2.507 ± 0.152	2.511 ± 0.146	<i>2.326 ± 0.145</i>	2.332 ± 0.153
30 days	3.354 ± 0.277	3.340 ± 0.282	3.233 ± 0.287	<i>3.229 ± 0.286</i>
90 days	5.587 ± 0.312	5.576 ± 0.318	<i>5.060 ± 0.310</i>	5.064 ± 0.310
180 days	8.296 ± 0.444	8.565 ± 0.452	7.298 ± 0.438	<i>7.282 ± 0.438</i>
365 days	17.258 ± 0.919	17.304 ± 0.895	<i>14.098 ± 0.829</i>	14.145 ± 0.828
730 days	18.270 ± 1.003	19.052 ± 1.042	16.396 ± 0.974	<i>16.395 ± 0.991</i>
\sum	8.003 ± 0.245	8.133 ± 0.248	<i>7.026 ± 0.225</i>	7.031 ± 0.226
4 weeks	9.483 ± 1.152	9.653 ± 1.198	7.941 ± 1.024	<i>7.867 ± 1.000</i>
13 weeks	9.365 ± 1.130	8.910 ± 1.047	<i>8.481 ± 1.076</i>	8.637 ± 1.222
26 weeks	8.895 ± 4.395	8.594 ± 4.404	8.184 ± 4.378	<i>8.092 ± 4.371</i>
52 weeks	13.340 ± 4.619	12.575 ± 4.036	<i>10.763 ± 3.519</i>	10.922 ± 3.784
104 weeks	17.452 ± 5.277	18.198 ± 6.037	16.273 ± 4.767	<i>16.008 ± 4.855</i>
\sum	10.690 ± 1.126	10.534 ± 1.127	<i>9.366 ± 1.010</i>	9.374 ± 1.041
6 months	12.150 ± 0.278	12.147 ± 0.286	9.158 ± 0.214	<i>9.096 ± 0.212</i>
12 months	12.183 ± 0.246	12.874 ± 0.269	10.479 ± 0.222	<i>10.420 ± 0.220</i>
24 months	9.544 ± 0.352	9.325 ± 0.353	<i>8.451 ± 0.327</i>	8.486 ± 0.333
60 months	12.724 ± 0.604	14.780 ± 0.740	11.617 ± 0.583	<i>11.449 ± 0.567</i>
120 months	15.051 ± 5.820	13.656 ± 4.244	10.798 ± 3.371	<i>10.650 ± 3.329</i>
\sum	11.763 ± 0.161	12.192 ± 0.174	9.774 ± 0.140	<i>9.719 ± 0.139</i>
\sum	11.047 ± 0.139	11.414 ± 0.149	9.254 ± 0.121	<i>9.210 ± 0.120</i>

Table 5: Forecasting results according to MAAPE forecasting error. \sum denotes results over all horizons, the best result for each row is formatted in italic-bold.

Horizon	Theta	Comb	Simple	Weighted
15 days	2.520 ± 0.151	2.536 ± 0.153	2.335 ± 0.148	<i>2.335 ± 0.153</i>
30 days	3.312 ± 0.268	3.296 ± 0.269	3.185 ± 0.273	<i>3.183 ± 0.272</i>
90 days	5.538 ± 0.316	5.526 ± 0.318	<i>5.033 ± 0.317</i>	5.036 ± 0.317
180 days	8.047 ± 0.433	8.366 ± 0.444	7.161 ± 0.434	<i>7.142 ± 0.434</i>
365 days	15.431 ± 0.783	15.538 ± 0.772	<i>13.132 ± 0.766</i>	13.165 ± 0.765
730 days	17.696 ± 0.971	17.989 ± 0.975	16.306 ± 0.993	<i>16.286 ± 1.000</i>
\sum	7.622 ± 0.226	7.719 ± 0.228	6.842 ± 0.218	<i>6.842 ± 0.219</i>
4 weeks	8.750 ± 1.027	8.856 ± 1.061	7.570 ± 0.953	<i>7.514 ± 0.938</i>
13 weeks	9.475 ± 1.146	8.896 ± 1.057	<i>8.636 ± 1.142</i>	8.717 ± 1.207
26 weeks	8.471 ± 4.175	8.233 ± 4.190	7.945 ± 4.213	<i>7.849 ± 4.208</i>
52 weeks	12.084 ± 3.451	11.620 ± 3.374	<i>10.231 ± 3.223</i>	10.270 ± 3.320
104 weeks	15.683 ± 3.885	16.119 ± 4.218	15.251 ± 3.890	<i>14.979 ± 3.921</i>
\sum	10.090 ± 0.955	9.891 ± 0.962	9.091 ± 0.938	<i>9.064 ± 0.949</i>
6 months	11.059 ± 0.242	10.983 ± 0.243	8.892 ± 0.207	<i>8.840 ± 0.206</i>
12 months	12.239 ± 0.244	13.203 ± 0.273	10.648 ± 0.224	<i>10.593 ± 0.223</i>
24 months	9.119 ± 0.320	8.906 ± 0.319	<i>8.190 ± 0.306</i>	8.200 ± 0.307
60 months	12.500 ± 0.601	13.512 ± 0.601	11.330 ± 0.556	<i>11.197 ± 0.548</i>
120 months	11.883 ± 3.194	11.499 ± 2.941	9.389 ± 2.538	<i>9.341 ± 2.539</i>
\sum	11.289 ± 0.151	11.676 ± 0.158	9.658 ± 0.136	<i>9.607 ± 0.136</i>
\sum	10.589 ± 0.129	10.916 ± 0.135	9.123 ± 0.118	<i>9.082 ± 0.117</i>

Results by MAAPE (see Table 5) demonstrate that both Simple and Weighted variants of meta-learning outperform Theta and Comb techniques. Weighted slightly outperformed Simple variant for more than half (11 out of 16) horizons and also overall (see last row in Table 5).

Table 6: Forecasting results according to MASE forecasting error. \sum denotes results over all horizons, the best result for each row is formatted in italic-bold.

Horizon	Theta	Comb	Simple	Weighted
115 days	1.047 ± 0.050	1.056 ± 0.055	<i>0.963 ± 0.046</i>	0.964 ± 0.049
30 days	1.365 ± 0.076	1.362 ± 0.077	1.311 ± 0.081	<i>1.310 ± 0.081</i>
90 days	2.245 ± 0.106	2.239 ± 0.107	1.997 ± 0.102	<i>1.996 ± 0.101</i>
180 days	3.265 ± 0.200	3.334 ± 0.200	2.910 ± 0.202	<i>2.904 ± 0.202</i>
365 days	5.987 ± 0.309	6.132 ± 0.320	<i>5.053 ± 0.301</i>	5.060 ± 0.300
730 days	7.676 ± 0.521	7.798 ± 0.517	<i>6.747 ± 0.484</i>	6.761 ± 0.504
\sum	3.115 ± 0.098	3.161 ± 0.099	<i>2.750 ± 0.091</i>	2.751 ± 0.092
4 weeks	0.579 ± 0.077	0.592 ± 0.084	0.480 ± 0.070	<i>0.475 ± 0.068</i>
13 weeks	0.486 ± 0.052	0.476 ± 0.058	<i>0.426 ± 0.047</i>	0.432 ± 0.051
26 weeks	0.681 ± 0.324	0.680 ± 0.350	0.622 ± 0.347	<i>0.599 ± 0.340</i>
52 weeks	0.993 ± 0.299	0.967 ± 0.308	0.821 ± 0.282	<i>0.806 ± 0.286</i>
104 weeks	1.713 ± 0.731	1.689 ± 0.701	<i>1.564 ± 0.729</i>	1.587 ± 0.753
\sum	0.733 ± 0.103	0.728 ± 0.103	0.637 ± 0.101	<i>0.635 ± 0.103</i>
6 months	0.642 ± 0.011	0.637 ± 0.011	0.513 ± 0.010	<i>0.511 ± 0.010</i>
12 months	0.730 ± 0.013	0.750 ± 0.014	0.625 ± 0.012	<i>0.622 ± 0.012</i>
24 months	1.153 ± 0.029	1.118 ± 0.029	0.977 ± 0.027	<i>0.977 ± 0.027</i>
60 months	1.986 ± 0.099	2.428 ± 0.131	1.919 ± 0.221	<i>1.827 ± 0.130</i>
120 months	4.217 ± 1.153	4.025 ± 1.120	3.162 ± 0.982	<i>3.144 ± 0.989</i>
\sum	0.921 ± 0.015	0.968 ± 0.018	0.797 ± 0.026	<i>0.785 ± 0.017</i>
\sum	1.332 ± 0.023	1.378 ± 0.025	1.163 ± 0.028	<i>1.153 ± 0.023</i>

Results by MASE (see Table 6) demonstrate that both Simple and Weighted variants of meta-learning outperform Theta and Comb techniques. Weighted slightly outperformed Simple variant for more than half (12 out of 16) horizons and also overall (see last row in Table 5).

Forecasting errors showed an expected and consistent tendency to increase together with increasing forecasting horizon length. Interestingly, MASE errors were lowest overall for weekly whereas SMAPE and MAAPE for daily data. To summarize over all data types and horizons: among benchmark methods Theta tends to slightly outperform Comb and meta-learning approaches win over both benchmarks with Weighted variant as the best.

6 Conclusions

Extensive evaluation of the proposed meta-learning approach on micro-economic time series from M4 competition demonstrated that meta-learning is capable to outperform benchmark methods Theta and Comb. Best performance was achieved by pooling forecasts from assistant-recommended univariate time series models using weighted average with weights corresponding to reciprocal rank. Regression meta-learner model was more successful and had a better out-of-bag fit for A1 than for A2 assistant. Lower forecasting errors

were obtained using weighted variant of forecasting ensemble over the Theta method: 9.21% versus 11.05% by SMAPE, 9.08% versus 10.59% by MAAPE, 1.33 versus 1.15 by MASE. Considering a larger set of time series data for training meta-learner and exploring usefulness of feature engineering by sequence-to-sequence auto-encoder would be interesting directions for further research.

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